Practical Monte Carlo Analysis

Perry K. Parendo
651-230-3861
Perry@PerrysSolutions.com

Agenda

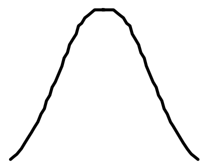
- **Monte Carlo Overview**
- **Monte Carlo Weaknesses**
- **Technical Approaches/ Case Studies**
  - **Vibration Design**
  - **Fatigue Strength**
  - **Dimensional**
  - **Can also be used with project schedules (not shown)**

Monte Carlo Overview

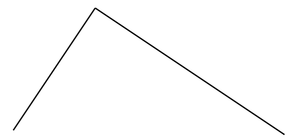
Premise

- **Assume A = B + C, we often insert fixed values and calculate**
  - **i.e. if B = 5 and C = 8, then A = 13**
- **In Monte Carlo Analysis, B and C are distributions and will estimate a distribution for A**
  - **If B is from 4-6 and C is from 6-9, within what window could A most likely be?**
  - **It would be rare to expect a value of 10 or 15, so those would not be a good expected value window.**

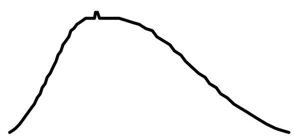
Distribution Options



Normal



Triangular



Skewed

Uniform

Process

- **Monte Carlo will randomly pick values from the probability distribution defined**
  - **For normal, more in middle and fewer near edges**
- **It will then apply the defined calculations**
- **This is repeated 1000s of times to create a distribution for the output**
  - **Typical is 10k to 15k, but ultimately we want the distribution to become stable**

Monte Carlo Weaknesses

- **Distribution**
  - **Not always normal**
- **Parameters**
  - **Will people know a standard deviation**
    - **Can they talk in that language**
  - **The more exotic the distribution, the more data required to define it**
  - **What if we only have a few prototype parts**
- **Model**
  - **How do we know our equation or model is right?**

Distribution Selection

- **We naturally tend to think of the normal distribution, yet things are often skewed. Mathematically, we can do normal and skewed**
- **Practically, no one talks about standard deviation – especially on new things or limited prototypes**
- **An easy and useful approach is to use the triangle distribution. We ask 3 simple questions:**
  - **What is the most likely value**
  - **What do you expect as the max (we tend to underestimate, so this actually works better mathematically)**
  - **What do you expect as the min (again, we do not go far enough)**

Model Building

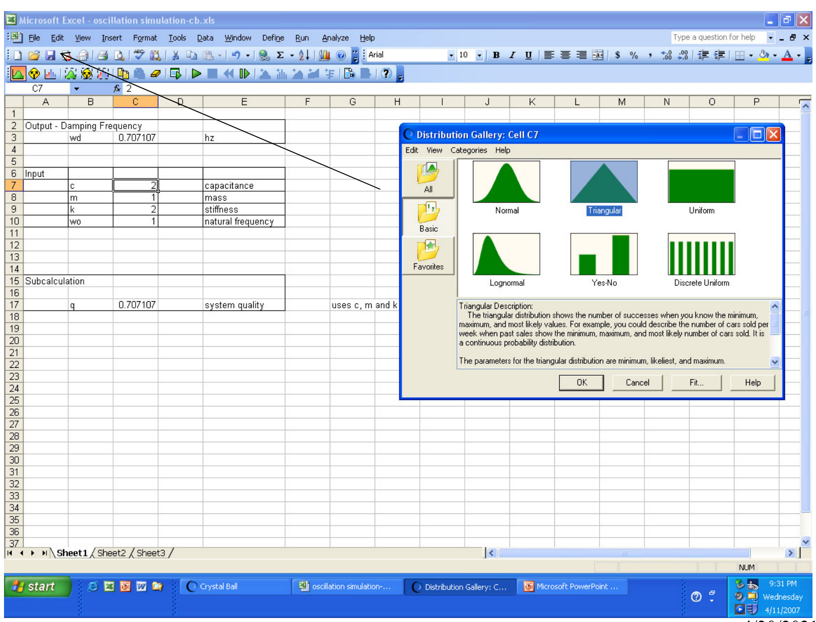
- **Reliability modeling is a common practice for understanding reliability requirements**
- **Product modeling is done to understand performance. How is product modeling done?**
  - **Solid models (does it fit together)**
  - **Simulations (scientific knowledge, how we expect it to work)**
  - **DOE (empirical – how it actually works)**
- **We can also have models for cost (development and unit), for schedule, and other non-performance areas.**
- **But what about modeling for uncertainty in these things?**

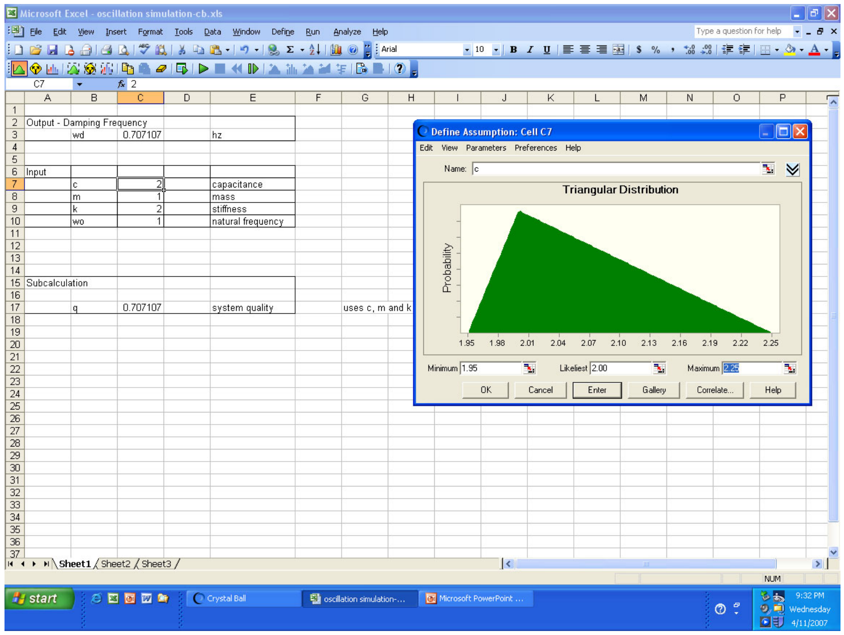
Model Building

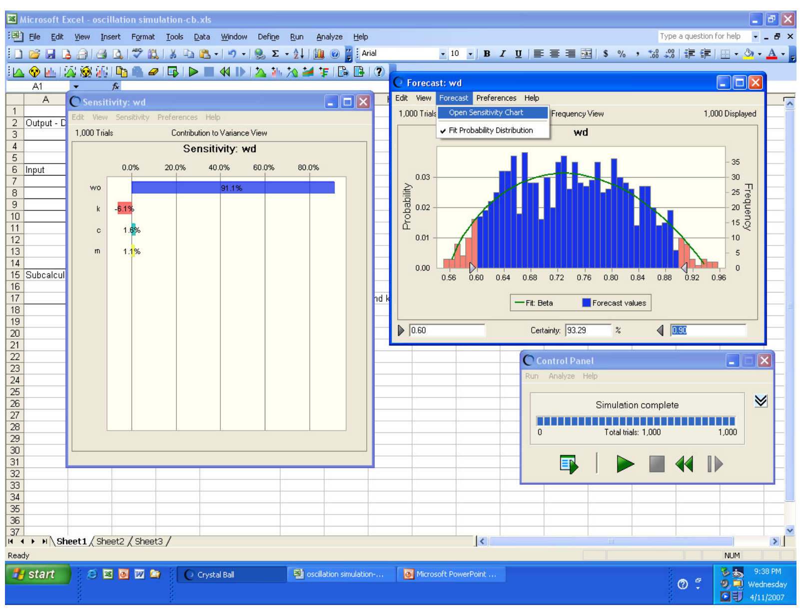
- **The purpose of several tools is to create an equation**
  - $Y = z + a * A + b * B + c*A*B + d*A^2 + e*B^2 + error$
- **This is from ANOVA, regression and DOE techniques**

Vibration Example

- **Goal: Obtain desired performance at lowest cost**
- **Response: Damping frequency and cost (tolerances as indicator)**
- **Approach: Determine drivers for uncertainty. Open tolerances for low impact items. Only tighten on drivers. Performance equation is known and input into Excel**
- **Result: Can evaluate impact of different confidence levels. Can open up tolerance on low impact items to save cost. Can obtain more information for areas with higher impact.**




Microsoft Excel - oscillation simulation-cb.xls
Output - Damping Frequency
wd 0.707107 hz
Input
c 2 capacitance
m 1 mass
k 2 stiffness
wo 1 natural frequency
Subcalculation
q 0.707107 system quality
uses c, m and k
Define Assumption: Cell C7
Triangular Distribution
Probability
Minimum 1.95
Likeliest 2.00
Maximum 2.25


Microsoft Excel - oscillation simulation-cb.xls
Sensitivity: wd
1,000 Trials
Contribution to Variance View
wo 91.1%
k -6.1%
c 1.8%
m 1.1%
Forecast: wd
Frequency View
1,000 Displayed
Probability
Frequency
Fit: Beta
Forecast values
Certainty: 93.29 %
Control Panel
Simulation complete
Total trials: 1,000

Structural Fatigue Case Study

- **Goal: Determine ability to meet life expectations**
- **Response: Fatigue strength exceeding load for at least 95% of units**
- **Approach: After resolving "obvious" issues, find a DOE sweet spot for the tunable variables.**
  - **Obvious issues resolved with solid models or simulations**
  - **Non-obvious issues are addressed with DOE**
- **Result: Determined areas of uncertainty that required deeper, specific information. Get development budget required. Understood we were capable of meeting the requirements, with the current conceptual approach**

Fatigue Equations

- **Fatigue equation for high cycles**
- **$S_f = 10^C * N^b$**
  - **Where N is number of cycles until failure**
  - **S_f is fatigue strength**
  - **$C = \log (0.8*S_{ut})2 / S_e$**
    - **S_{ut} is ultimate tensile strength**
    - **S_e is endurance limit**
  - **$b = -(\log(0.8S_{ut} / S_e) / 3$**
- **Example data: S_{ut} for G41300 steel is 106 ksi**
  - **S_e is 40 ksi with a standard deviation of 1.1 ksi**
  - **These are "maximum" values – see next page for modifiers**
  - **What if it is welded? These are not published – they are your capabilities. Need DOE to get the proper model**

Fatigue Modifiers

- S_e modifying factors may include:
- Surface factor – can range from 0.85 to 0.65 if machined (depends on S_{ut})
  - Usual roughness from 250-32, extreme from 1000-6 for milling
  - Could vary depending upon supplier or delivery schedule
- Temperature factor: 1 – 0.0032 * (T – 840) from 840 F to 1020 F
  - What temperatures will be experienced?
- Stress concentration factor: From 0.55 to 0.85 depending on notch radius (from 0.01 to 0.10 inches)
  - This is not just from manufacturing, but also from field

Fatigue Loading

- Is the applied load constant? Likely not – even if we ignore fluctuating stresses (different equations)
  - Are we driving our car or a rental car?
  - There is a distribution to this value also

Statistical Tolerancing Case Study

- Goal: Ensure low probability of failure for parts fitting together
- Response: Mechanical tolerance gap. Known sensitivity of installed sensor
- Approach: Using basic math for the gap, determine areas where uncertainty needs to be resolved
  - When uncertainty of inputs makes little difference, do not dig any deeper
- Result: Reduced expected failure rate to acceptable level. Tolerances relaxed where limited impact on performance, but that would reduce cost of producing part (and cycle time)

Tolerance Analysis Approaches

- Worst case
- Statistical
  - Root Sum of Squares is a subset of this area

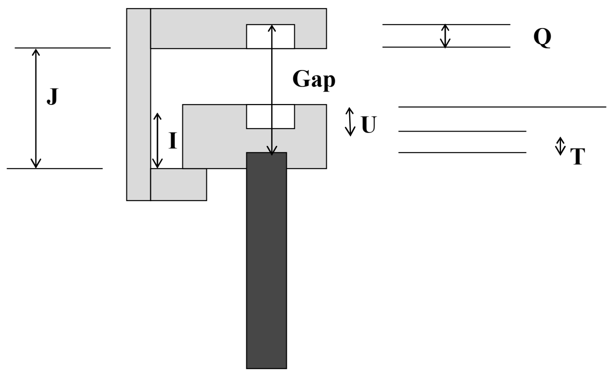
What is Worst Case

- In this situation, we look at the extremes
- This is a conservative approach
- Good for safety
- Not so good for cost

Gap Example

- What is the gap between the top indent and the functional surface of the dark part?
- Create the "gap" equation
- Apply tolerances from following tables
- What gap will we experience?
- Gap = Q + J – I + U + T
  - Diagram on next page

Gap Graphic



Practical MC Analysis | Perry's Solutions, LLC | 4/20/2021

Statistical Tolerancing

- **Requires:**
- **Known relationship between inputs and outputs**
  - **For tolerancing of part fit, it is a simple sum**
  - **For general situations, it can be a complex relationship**
- **Known distribution of inputs**
- **Known distribution parameters**
- **Does not require normal distribution of input parameters**

Practical MC Analysis | Perry's Solutions, LLC | 4/20/2021

Statistical Tolerancing

- **Reality:**
- **Maybe you know the process capability**
  - **Lots of prototypes**
  - **History with vendor and similar components**
- **Maybe you only have a small sample of parts**
  - **A triangular distribution can work well**
  - **"Most likely" – peak of triangle**
  - **Lowest expected**
  - **Highest expected**

Capability Determination - Statistical

Feature	Material/ spec	Capability – cp	Typical Range, inches	Source	Basis	Distribution
J – latch on part A	Plastic, 3.538 +/- 0.005		+/- 0.004	Mary	Vendor communication	Triangle, balanced
Q – indent	Plastic, 0.050 +/- 0.005		Full	Mary	Vendor communication	Skewed, peak is 1/3 of way down
I – latch on part B	Metal, 1.673 +/- 0.005		Full	Bob	Similar product	Normal
T – installation of assy, to be flush	Threaded, 0.000 + 0.000 / - 0.010		0 to 0.008	Sue	Prototype experience	Skewed, at 0.001 as peak
U – second indent	Metal, 0.035 +/- 0.005		+/- 0.003	Bob	Similar processes	Triangle, balanced

Conclusion

- **Simple language and tools can help ease new people and projects into thinking about variability**
- **Iterative Monte Carlo Modeling is a way to address weaknesses while answering questions**
- **Early DOE work can provide the foundation for advanced modeling needs**
- **If you want to see reference materials – visit our website**
  - **www.PerrysSolutions.com**
  - **If interested, email us to be on our quarterly newsletter where we share recent trends and learning points**
    - **Newsletters are all archived on our publications page**